Arithmetic Testing OnLine (ATOL)ᔆᴹ Assessment Framework

Overview

Assessment Objectives (AOs) are used to describe the arithmetic knowledge and skills that should be mastered by the end of each year in grades 3 - 6. The expectations are designed to prepare a typical student to excel at the middle and upper grades in a challenging, college-preparatory mathematics curriculum. A full range of problem types are assessed. An AO designated with a "**W**" in the "Type" column may be assessed either by a *Word Problem* or by a problem calling for straight computation. An AO designated win an **"F"** indicates that the skill is to be assessed as a *Math Fact*.

Word Problems (**W**)
  a. Requires the student to discern which operations to use and their proper order of application;
  b. Unless noted otherwise, the word problem does not contain extraneous information;
  c. Involves a relatively direct translation of the problem to a mathematical solution.

Math Facts (**F**)
  a. Measures automaticity or fluency in execution and assumes thorough memorization;
  b. Test items are time-limited such that only a quick response will be successful;
  c. Involve a full range of facts that require "fluency", such as prime numbers, multiplication tables, and commonly used fraction to decimal conversions.

Each grade level in ATOLᔆᴹ includes problems with a range of complexity and number of steps. This range is categorized into three levels of cognitive complexity. AOs marked with more than one level of complexity may be assessed by multiple test questions reflecting this range.

Cognitive Complexity:

Level 1: Complexity is the lowest, and usually involves a simple recall of well-learned math facts; e.g., memorization of the multiplication tables to full *automaticity*.

Level 2: Complexity involves straight-forward computational knowledge, if word problems are involved, the translation to mathematical computation is very straightforward, requiring few steps in the process.

Level 3: In word problems, complexity involves a translation of more complex statements that may contain redundant, or irrelevant information, or may require several steps to solution. Conceptual complexity involving core math principles may also be present.

Note:
The numbers in the left hand column (e.g., 3.12) indicate the assessment objective. The first number corresponds to the grade level (e.g., 3.12 is a third grade objective).

GRADE 3	Type	Cognitive Complexity		
Unless noted otherwise, in Third Grade, numbers are greater than or equal to zero, or less than or equal to 10,000; fraction denominators that do not exceed 8.	**W** = Word problems **F** = Math Facts	1	2	3

NUMBER LINE, ORDERING & MAGNITUDE

		Type	1	2	3
3.01	Compare and order whole numbers and place whole numbers on a marked number line. *Content Limits*: Unless an option is incorrect, the ordering of numbers is from left to right, with smaller numbers to the left. Number lines can include zero.		✔		
3.02	Given a number in place value notation, write or identify the number in expanded notation*, or vice versa, (e.g., 235 = 2x100 + 3x10 + 5x1) or determine the magnitude represented by any digit in the place value representation of a whole number. *Content Limits*: Further example: 235 = 100x2 + 10x3 + 1x5, is acceptable. However, do not alter the order of the sums from standard place value positions; e.g., 68 = 8x1 + 6x10, would be prohibited. No decimal numbers. *Expanded notation: Expressing a number as the sum of products of 10^n (e.g., 523 = 5x100 + 2x10 + 3x1.).			✔	
3.03	Compare two unit* fractions with different denominators. * Unit fraction: A fraction with 1 in the numerator; e.g., $\frac{1}{3}$.			✔	
3.04	Compare two fractions with compatible denominators* using figures involving areas or lengths (e.g., $\frac{2}{5} < \frac{3}{5}$.) *Content Limits*: Figures using areas should use only rectangular figures. Use only fractions with equal denominators. * Compatible denominators: Denominators that are equal, or where one denominator is a small positive multiple (e.g., x2 or x3) of the other.			✔	
3.05	Round a whole number to the nearest 10s, 100s, or 1000s. *Content Limits*: Do not include problems requiring rounding from 5.	**W**		✔	

3.06	Convert between decimal notation and fractions in simple money problems; (e.g., $\frac{1}{2}$ dollar equals \$0.50.) (Illustrates the idea that decimals and fractions provide different ways to express the same quantity.) *Content Limits*: Some items must be word problems.	**W**		✔	
3.07	Write or select simple fractions that are equivalent; (e.g., $\frac{4}{6} = \frac{2}{3}$; $\frac{1}{2} = \frac{3}{5}$) *Content Limits*: No additional limits.			✔	
3.08	Add or subtract two fractions less than one with compatible denominators where the sum is less than or equal to one or the difference is non-negative; (e.g., $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$; $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$) *Content Limits*: Note: non-negative numbers are greater than or equal to zero.			✔	

FACTORING

3.09	Select a number from a list that requires counting by twos, fives, or tens; (e.g., "Which number is a multiple of two?") *Content Limits*: Limit whole numbers to less than 100.		✔		

OPERATIONS

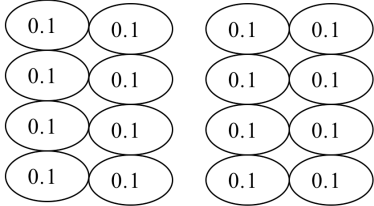|					
3.10	Solve addition, subtraction, multiplication, or division problems that use the special properties of 0 or 1. *Content Limits*: Do not include items that express division by 0. Can be rolled into another AO; e.g., 3.02.	**F &W**	✔	✔	
3.11	Recall simple addition and subtraction facts fluently; (e.g., 15 + 4 = ? ; 18 - 3 = ?) *Content Limits*: Task is timed, so the response mode should require very little time. A single item may involve multiple problems. Items should not involve numbers greater than 100, and should not involve either carrying or borrowing, unless the action is trivial (e.g., 49 + 1 = ?) See appendix notes for additional information.	**F**	✔		

3.12	Find the sum of two four-digit whole numbers whose sum is less than 10,000 and requires carrying, and the difference of two three-digit whole numbers, less than 5,000, that requires borrowing. *Content Limits*: No additional limits.	**W**		✔	✔
3.13	Find the sum of three two-digit numbers. *Content Limits*: No additional limits.	**W**		✔	✔
3.14	Recall multiplication facts fluently through 10 x 10. *Content Limits*: Task is timed, so the response mode should require very little time. See appendix notes for additional information.	**F**	✔		
3.15	Find the product of a two- or three-digit whole number by a single digit whole number. *Content Limits*: No additional limits.	**W**		✔	✔
3.16	Find the quotient of a two- or three-digit whole number by a single digit whole number--no remainders. *Content Limits*: No additional limits.	**W**		✔	
3.17	Check an addition/multiplication problem by using subtraction/division, and vice-versa. *Content Limits*: No additional limits.	**W**		✔	
3.18	Solve money problems of the form: given two of the three quantities--either total cost, unit cost, or unit numbers--find the third. *Content Limits*: No additional limits.	**W**		✔	✔

STANDARD ALGORITHMS

3.19	Identify or write the next step in an addition or subtraction problem involving carrying or borrowing using the standard algorithm with two multi-digit whole numbers. *Content Limits*: No decimals. See appendix notes for additional information.			✔	
3.20	Identify or write the next step in a multiplication problem using the standard algorithm of one multi-digit whole number multiplied by a single digit whole number. *Content Limits*: No decimals. Problems are primitive.			✔	

GRADE 4 *Content Limits*: Unless noted otherwise, in 4th grade, numbers are non-negative numbers less than or equal to 1,000,000, fractions with denominators that can be any whole number up to and including 12; decimals are to two places.	Type **W** = Word problems **F** = Math Facts	Cognitive Complexity		
		1	2	3
NUMBER LINE, ORDERING & MAGNITUDE				
4.01 Compare and order whole numbers, less than one million, and decimals to two places, using symbols <, =, and > or by placement on a marked number line. *Content Limits*: Extends 3.01 with the required use of symbols. If decimals are used, limit number size to four digits; e.g., 10.55. Number lines include zero.		✔		
4.02 Use the meaning of place value notation to estimate the sum or difference of two multi-digit numbers. *Content Limits*: Do not group numbers on top of each other (e.g., $\frac{1}{3}$); numbers must exceed 4 digits in order to tap estimation skills and avoid the use of simple addition to derive a solution. Do not use decimals.	**W**		✔	
4.03 Locate the position of common fractions, decimals to tenths, and mixed numbers (non-negatives only) on a marked number line; (e.g., 0.2, $\frac{3}{4}$, 0.7, 1.6, $1\frac{3}{5}$, etc..) *Content Limits*: Number lines include zero.			✔	
4.04 Round decimals to the nearest tenths place. *Content Limits*: Do not include problems requiring rounding with numbers ending in 5.	**W**		✔	
4.05 Round a whole number to the nearest 10s, 100s, 1000s, or 10000s. *Content Limits*: Do not include problems requiring rounding from 5.	**W**		✔	

FRACTIONS, DECIMALS, PERCENTS

4.06	Know and use decimal representations and fractional equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, and $\frac{1}{100}$ (especially with money.) (e.g., $0.3 = \frac{3}{10}$.) *Content Limits*: AO can be incorporated into word problems.	**W**		✔	
4.07	Evaluate a decimal expression involving multiplication that reflects a picture of the relationship between multiplication and division; (e.g., 0.4 x 4 = 1.6 corresponds to a picture using 4 columns and 4 rows of 0.1.) *Content Limits*: Illustrations must be carefully considered. 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 Figure can represent 2 x 0.8, 4 x 0.4, 8 x 0.2, or 16 x 0.1, depending upon partition placement. See appendix notes for additional information.				✔
4.08	Identify or write a fraction in lowest terms. *Content Limits*: No additional limits.	**W**		✔	
4.09	Write a whole number as a fraction, (e.g., $2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \dots$) or express a part-whole relationship as a fraction; (e.g., "If Mary completed 16 out of 20 steps, what fractional part of the 20 steps did she complete? Answer in lowest terms.") *Content Limits*: No additional limits.	**W**		✔	

4.10	Write the sum of two fractions with compatible denominators and sum greater than or equal to 1 as an improper fraction (e.g., $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$; $\frac{3}{4}$ $+ ? = \frac{5}{4}$) *Content Limits*: Sums should not exceed 2. Compatible denominators are limited to multiples of 1, 2 and 3.			✔	
4.11	Write the sum of two fractions with compatible denominators and sum greater than or equal to 1 as a mixed number; (e.g., $\frac{5}{5} + \frac{3}{5} = 1 + \frac{3}{5} = 1\frac{3}{5}$; $1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3} = 1\frac{1}{3}$; $\frac{2}{3} + \frac{4}{6} = 1\frac{1}{3}$ $\frac{4}{7} + \frac{5}{7} = \frac{9}{7} = 1\frac{2}{7}$). *Content Limits*: Sums should not exceed 2.		✔	✔	

FACTORING

4.12	Perform an operation such as selecting a number from a list that requires counting by threes or fours; (e.g., "Which number is a multiple of three?") *Content Limits*: Illustrations may be used, such as a picture of several cows showing their heads and their feet, with the fence blocking the view of their bodies. Multiples of two can involve numbers as large as 1,000, multiples of threes and fours should involve numbers less than 50.		✔		
4.13	From a list of fractions, identify two equivalent fractions. *Content Limits*: Item complexity should clearly extend the skill in 3.07.		✔		

OPERATIONS

4.14	Solve addition, subtraction, multiplication or division problems that use the special properties of 0 or 1. *Content Limits*: Do not include items that express division by 0.	**F&W**	✔		
4.15	Recall multiplication and division facts fluently (through 10 x 10 and 100 ÷ 10) and include multiplication facts that assess place value knowledge (e.g., 274 x 10 = ?) *Content Limits*: Division facts include only whole number divisions that do not yield remainders; (e.g., 24 ÷ 3 = ?; 18 ÷ 2 = ?)	**F**	✔		

4.16	Find the sum or difference of two whole numbers and decimal numbers that requires carrying or borrowing up to ten-thousands place, and down to one-hundredths place. (e.g., 1356 + 188 = ?; 10.07 - 6.22 = ?) *Content Limits*: At least one item should add two five-digit numbers. For information on math facts, see appendix notes for additional information.	**F&W**	✔	✔	✔
4.17	Find the sum of four two-digit numbers and three four-digit numbers. *Content Limits*: Whole numbers only. Requires carrying.			✔	✔
4.18	Find the product of two whole numbers up to three digits each. *Content Limits*: No additional limits.	**W**		✔	✔
4.19	Find the quotient of a three-digit whole number divided by a two-digit number using long division, with remainders. *Content Limits*: Remainders are written as R = ?.			✔	
4.20	Express a whole number as a product of a given whole number with another whole number summed with a whole number remainder that is less than the given number; (e.g., "There are 28 students going to a museum. They will travel in a number of cars. Each car can seat five students, and only one car will not be filled with five students. How many students will be in the remaining car? (And as a follow-up question.) How many of these cars will carry five students?") *Content Limits*: No additional limits.	**W**			✔
4.21	Check a whole number division problem with remainder by multiplying the quotient by the divisor and subtracting the result from the dividend. *Content Limits*: No additional limits.			✔	

STANDARD ALGORITHMS

				✔	
4.22	Fill in a missing step in an addition or subtraction problem using the standard algorithm with two multi-digit whole numbers. *Content Limits*: Digits do not exceed four digits.			✔	
4.23	Fill in a missing step in a multiplication problem using the standard algorithm for one multi-digit whole number multiplied by a two digit whole number. *Content Limits*: Digits do not exceed three digits.			✔	
4.24	Fill in a missing step in a long division problem with a multi-digit dividend and single digit divisor. *Content Limits*: Digits do not exceed three digits. No remainders.			✔	

GRADE 5	Type W = Word problems F = Math Facts	Cognitive Complexity		
Content Limits: Unless noted otherwise, in 5th grade, numbers are non-negative and fraction denominators can be any whole number value up to 20.		1	2	3

NUMBER LINE, ORDERING & MAGNITUDE

5.01	Place numbers including mixed numbers, improper fractions, and decimals on a marked number line; (e.g., 0.27, $\frac{3}{16}$, 3.14, $\frac{32}{15}$, etc.) *Content Limits*: Extends 4.01 with decimals to thousandths; e.g., 85.557. See 5.04 for one illustration. Number line must include zero.			✔	
5.02	Write and order whole numbers in place value or expanded notation, through one million. To a prescribed degree of accuracy, estimate the magnitude of a number in place value or expanded notation through one million. *Content Limits*: No additional limits.		✔		
5.03	To a prescribed degree of accuracy, estimate the sum or difference of decimals, fractions, and mixed numbers or the product (or quotient) of two whole numbers; (e.g., 0.10 + 0.01 is closer to 0.1 than to 0.2; 23 x 300 is closer to 7000 than 6000; 15/4 is closer to 4 than to 3.) *Content Limits*: No additional limits.	W		✔	
5.04	Round a whole number to the nearest 10s, 100s, 1000s, 10000s, or 100000s place, and decimals to the nearest one hundredths place. *Content Limits*: Do not include problems requiring rounding from 5. Can be combined with 5.01 by rounding a decimal up; e.g., on a marked number line showing 85.00 and 85.01 on either end, round 85.007 to 85.01.	W		✔	

FRACTIONS, DECIMALS, PERCENTS

5.05	Convert between percent, decimal and fraction equivalents; (e.g., $\frac{3}{5}=60\%=0.6$; $1.5=150\%$; $\frac{3}{5}=\frac{6}{10}=0.3\overline{3}$; $\frac{10}{15}=\frac{6}{9}$; or, "Larry spent \$0.50 of the \$1.50 he had. What fraction of his money did he spend?") *Content Limits*: No additional limits.	**W**		✔	✔
5.06	Convert between mixed numbers and decimals; (e.g., $5\frac{3}{4}=5.75$). *Content Limits*: Decimals should only have two places.			✔	
5.07	Evaluate a decimal expression involving division by using a picture of the relationship between multiplication and division; (e.g., "$3\overline{)0.18}=$" corresponds to a model of 9 columns and 2 rows of 0.01s partitioned into groups of 3.) *Content Limits*: No additional limits.				✔
5.08	Find the sum or difference of two mixed numbers with different denominators. *Content Limits*: No additional limits.	**W**		✔	✔
5.09	Show fractional equivalents; (e.g., "$\frac{5}{8}$ is the same as which number: $\frac{5}{8}, \frac{12}{20}, \frac{10}{16}, \frac{13}{20}$?") *Content Limits*: No additional limits.			✔	
5.10	Find the sums or differences of fractions and mixed numbers with like and unlike denominators. *Content Limits*: Items can include operations with a fraction and a mixed number, or two mixed numbers.	**W**		✔	
5.11	Find the product of two non-negative fractions.			✔	
5.12	Solve problems containing decimals, fractions, percents, and/or mixed numbers (e.g., "Jim began the day with \$15 and spent 20% on a ticket to the ball game, 25% for the bus trips and 15% on candy. How much money did he have left to spend?") *Content Limits*: No additional limits.	**W**			✔

FACTORING

5.13	Identify the prime numbers less than 50. *Content Limits*: Task is timed, so the response mode should require very little time.	**F**	✔		

5.14	Determine the prime factorization* for any whole number up to 50. 　　*Content Limits*: Do not use powers. * A prime factorization is the factorization (division) of a number into its prime number components (e.g., 8 = 2 x 2 x 2, is a prime factorization, whereas 8 = 4 x 2 is not, since 4 is not a prime number.)				✔	
5.15	Find the Least Common Multiple* of two composite numbers** less than 20. 　* Least Common Multiple is the smallest common product of two whole numbers; (e.g., common multiples of 2 and 3 are 6, 12, 18, etc., the Least Common Multiple of 2 and 3 is 6. The LCM of 5 and 12 is 60 and the LCM of 10 and 15 is 30.) 　** Composite numbers are positive whole numbers that are divisible by two or more distinct whole numbers greater than 1.				✔	

OPERATIONS

5.16	Demonstrate memorization of simple quotients without the need of paper and pencil computation; $$(e.g., \frac{2}{4} = \frac{1}{2}, \frac{2}{8} = \frac{1}{4}, \frac{6}{3} = 2; \frac{50}{10} = 5, etc..)$$ 　　*Content Limits*: Task is timed, so the response mode should require very little time.	**F**	✔			
5.17	Compute the sum of three whole numbers up to the ten thousands place and decimals down to one-thousandths. 　　*Content Limits*: No additional limits.	**W**			✔	✔
5.18	Compute the difference of two whole numbers up to the thousands place and decimals down to one-thousandths involving repeated borrowing, with zeros. 　　*Content Limits*: E.g., 345 - 287 = ?; 　　0.90 – 0.876 = ?. No additional limits.	**W**			✔	✔
5.19	Multiply and divide whole numbers and decimal numbers by 10, 100, and 1000. 　　*Content Limits*: Task is timed, so the response mode should require very little time.	**F**	✔			
5.20	Find the product of two (at most) four digit numbers. 　　*Content Limits*: Do not use decimals. Do not use one-digit multipliers.	**W**			✔	✔
5.21	Find the quotient of a two- or three-digit number by a two-digit number using long division. Write the answer in quotient, remainder form or as a mixed number. 　　*Content Limits*: Do not use decimals in dividend or divisor.	**W**			✔	✔

STANDARD ALGORITHMS

5.22	Identify an incorrect step in an addition or subtraction problem using the standard algorithm with two multi-digit whole numbers. 　　*Content Limits*: Do not use decimals.				✔	
5.23	Fill in a missing step in a multiplication of a multi-digit whole number and a decimal using the standard algorithm. 　　*Content Limits*: No additional limits.				✔	

| 5.24 | Fill in a missing step in a long division computation of a decimal by a whole number.
Content Limits: Remainders are expressed as R=? | | | ✔ | |

GRADE 6 *Content Limits*: Rational numbers which include negative numbers are introduced. Unless noted otherwise, in 6th grade, negative numbers are introduced on the number line although they are not yet used in operations other than computing distance on the number line. All operations yield positive numbers, and exponents are positive whole numbers.	Type **W** = Word problems **F** = Math Facts	Cognitive Complexity		
		1	2	3

NUMBER LINE, ORDERING & MAGNITUDE

		Type	1	2	3
6.01	Place non-negative and negative numbers on a marked number line, including integers, mixed numbers, improper fractions, decimals and powers of two. *Content Limits*: No additional limits.		✔		
6.02	Compare by ordering non-negative and negative integers, decimals to two places, fractions, and mixed numbers using symbols. *Content Limits*: Use one of each of the five number types.		✔		
6.03	Write a number using place value or expanded notation. *Content Limits*: Numbers can range between one billion and one-thousandths (inclusive.) E.g., "Write one hundred million in place value notation."			✔	

6.04	Estimate the product or quotient of two decimal numbers to a prescribed degree of accuracy;
	    (e.g., $10\overline{)24.5}$ is closer to 2 than to 3.)
	    Content Limits: No additional limits.
		W		✔	✔

6.05	Place the absolute value of a negative integer on a marked number line. *Content Limits*: Select between several numbers where l*a*l and l-*a*l and *a* are all among the options.			✔	
6.06	Compute the distance between two numbers. *Content Limits*: Do not rely upon a specific unit of measure, hence, computations should be made with numbers on the number line. Some number pairs for this problem can both be negative, some can both be positive, and some can have one positive and one negative number.			✔	

FRACTIONS, DECIMALS, PERCENTS

6.07	Convert between any two standard number representations including percents, decimals and fractions; (e.g., $\frac{2}{3} = 37.5\% = 0.375;\ -\frac{3}{5} = -0.60;$ etc..) *Content Limits*: No additional limits.	**W**		✔	✔
6.08	Recognize the common equivalents ($\frac{1}{8} = 0.125$) and common repeating decimal equivalents ($\frac{1}{3} = 0.3\bar{3}$ etc..) *Content Limits*: Common equivalents include all decimal equivalents of the unit fractions through 1/12. Task is timed, so the response mode should require very little time.	**F**	✔		
6.09	Find the percentage of two quantities; (e.g., "Find what percentage 35 cents is of $2.00;" or, "5 percent of $200.00 is…") *Content Limits*: No additional limits.	**W**		✔	✔
6.10	Solve problems with proportions; (e.g., "Solve $\frac{x}{20} = \frac{2}{10}$ for *x*.") *Content Limits*: No additional limits.	**W**		✔	✔
6.11	Know that ratios can be used to make comparisons of relative size*; (e.g., "The red car goes 20 miles per hour for 1/3 hour, and the blue car goes 20 miles per hour for 2/3 hour. How much farther does the blue car go than the red car?" Or: "Mary's 30 hits for 90 at bats is the same batting average as Peter's 10 hits for how many at bats?") * Comparisons of relative size follows from the fact that ratios remain the same under scaling of both quantities.	**W**		✔	✔

FACTORING

6.12	Determine the Least Common Multiple and the Greatest Common Factor* of two numbers. Use this to find a common denominator in fraction problems, and to find the lowest terms for a fraction. * The Greatest Common Factor is also known as the Greatest Common Divisor, which is the largest whole number divisor common to two whole numbers. (A divisor is a whole number that evenly divides another whole number.)	**W**		✔	✔
6.13	Perform computations involving the distributive property and evaluate expressions involving parentheses. *Content Limits*: Use only non-negative integers.			✔	

OPERATIONS

6.14	Find the quotient of a multi-digit number including decimals divided by a three digit whole number or a decimal (to one-hundredths) using long division; (e.g., $2.45\overline{)813.4} = ?$; $254\overline{)13.23} = ?;$) *Content Limits*: In at least one problem express quotient as a mixed number.	**W**		✔	✔
6.15	Find the product or quotient of two non-negative fractions. *Content Limits*: No additional limits.	**W**		✔	
6.16	Find the difference of two numbers up to thousands place (and/or down to thousandths place for decimals) involving borrowing over multiple zeros; (e.g., 500.04 - 48.651 = ?) *Content Limits*: No additional limits.	**W**		✔	✔
6.17	Find the product of two numbers up to thousands place and/or down to thousandths place for decimals; (e.g., 124.5 x 12.48 = ?) *Content Limits*: No additional limits.	**W**		✔	✔

STANDARD ALGORITHMS

6.18	Fill in a missing step in a multiplication of two multi-digit decimals using the standard algorithm. *Content Limits*: No additional limits.			✔	
6.19	Fill in a missing step in a long division of two decimals using the standard algorithm. *Content Limits*: No additional limits.			✔	